CERTIFIED PUBLIC ACCOUNTANT

FOUNDATION LEVEL 1 EXAMINATION

F1.1: BUSINESS MATHEMATICS AND QUANTITATIVE METHODS

DATE: THURSDAY,27 APRIL 2023

MARKING GUIDE AND MODEL ANSWERS

QUESTION ONE

Marking Guide

Kigali Ltd (KL)	***Marks***
a) (i) Allocation on the table (1 Mark each, max 6)	6
Optimal transportation schedule	1
(ii) Minimum transportation cost	1
Maximum marks	**8**
b) (i) Linear programming model	
Stating the decision variables	1
Describing the objective function	0.5
Defining the constraints (0.5 Marks, max 1.5)	1.5
Maximum marks	**3**
(ii) Solution of the linear problem using simplex method	
Insertion of slack variables into the constraints	1
Insertion of slack variables in the objective function	0.5
Drawing simplex tables (1 Mark each, max 5)	5
Decision on the number of carpets produced	0.5
Maximum marks	**7**
(iii) Application of linear programming (1 Mark each, max 2)	2
Total marks	**20**

Model Answers

a) i)Vogel's Approximation Method

Step 1: Identify the two lowest costs in each row and column of the given cost matrix and then write the absolute row and column difference. These differences are called penalties.

Step 2: Identify the row or column with the maximum penalty and assign the corresponding cell's min (supply, demand). If two or more columns or rows have the same maximum penalty, then we can choose one among them as per our convenience.

Step 3: If the assignment in the previous satisfies the supply at the origin, delete the corresponding row. If it satisfies the demand at that destination, delete the corresponding column.

Step 4: Stop the procedure if supply at each origin is 0, i.e., every supply is exhausted, and demand at each destination is 0, i.e., every demand is satisfying. If not, repeat the above steps, i.e., from step 1.

Initial Table

Branches	**Customers**				
	A	B	C	D	**Availability**
Musanze (M)	1,300	1,100	1,500	2,000	300
Nyagatare (N)	1,700	1,400	1,200	1,300	400
Huye (H)	1,800	1,800	1,500	1,200	200
Requirements	200	300	300	100	900

i) Table 1 Calculation of penalty and first allocation

Branches	**Customers**				Availability	Penalty
	A	B	C	D		
M	1,300 (200)	1,100	1,500	2,000	**300/100**	200
N	1,700	1,400	1,200	1,300	**400**	100
H	1,800	1,800	1,500	1,200	**200**	300
Requirements	**200/0**	**300**	**300**	**100**		
Penalty	400/0	300	300	100		

Highest penalty is 400 and Customer A are satisfied and only 100 computers remain in Branch M.

Table 2 Calculation of new penalty and Second allocation

Branches	**Customers**			**Availability**	**Penalty**
	B	**C**	**D**		
M	1,100(100)	1,500	2,000	100	400
N	1,400	1,200	1,300	400	100
H	1,800	1,500	1,200	200	300
Requirements	**300/200**	**300**	**100**		
Penalty	300	300	100		

Highest penalty is 400 for branch M. all 100 computers remain in Branch M will be sold to customer B and pending computer will be taken to other branches for the next dispatch.

Table 3 Calculation of new penalty and Third allocation

Branches	**Customers**			**Availability**	**Penalty**
	B	**C**	**D**		
N	1400 (200)	1,200	1,300	**400/200**	100
H	1,800	1,500	1,200	**200**	300
Requirements	**200**	**300**	**100**		
Penalty	400	300	100		

Highest penalty is 400 and Customer B are satisfied and only 200 computers remain in Branch N.

Table 4 Calculation of new penalty and fourth allocation

Branches	**Customers**		**Availability**	**Penalty**
	C	**D**		
N	1200(200)	1,300	**200**	100
H	1,500	1,200	**200**	300
Requirements	**300/200**	**100**		
Penalty	300	100		

Highest penalty is 300 for branch H and Customer C so it is up to candidate to choose where to start. For this let us start with penalty for customer. all 200 computers remain in Branch N will be sold to customer C and pending computer will be taken to other branches for the next dispatch.

Table 5 Calculation of new penalty and fifth allocation

Branches	**Customers**		**Availability**	**Penalty**
	C	D		
H	1,500	1,200 (100)	**200**	300
Requirements	**100**	**100**		

Table 6 Calculation of new penalty and fifth allocation

Branches	**Customers**	**Availability**
	C	
H	1,500 (100)	**100**
Requirements	**100**	

The optimal transportation schedule is that Muhanga branch supplies customer A and B, Nyagatare supplies customers B and C while Huye supplies customers C and D

ii) Minimum transportation cost = (200*1,300) + (100*1,100) +(200*1,400) +(200*1,200) +(100*1,500) +(100*1,200) = FRW 1,160,000

b)

i) Linear programming problem for BL

- Decision variables: Let x represent the number of carpets of X type and y be the number of carpets of Y type
- Objective function: The objective is maximizing profit
  Profit (P) = 4,000 x + 6,000y (in FRW)
- Constraint:

Subject to: $10x + 16y \leq 6,000$ Material constraint – wool

$10x + 8y \leq 3,600$ (60 minutes*60 hours) Labor constraint

$x \geq 0, y \geq 0$ Non – negativity constraint

ii) Solution of linear programming problem using simplex method

Insertion of slack variables in the constraints and objective function

$10x + 16y + 1S_1 = 6{,}000$ Material constraint

$10x + 8y + 1S_2 = 3{,}600$ Material constraint

$P - 4{,}000x - 6{,}000y + 0S_1 + 0S_2 = 0$

Initial simplex tableau

	X	Y	S_1	S_2	P	Solution
S_1	10	16	1	0	0	6,000
S_2	10	8	0	1	0	3,600
P	– 4,000	– 6,000	0	0	1	0

Taking the most negative value in the last row of the initial table, we find the pivot number as is 16 since 6000/16 =375 and 3600/8=450. We therefore take the smaller value which is 375.

Table 2. Divide all the elements in the first row by 16 because it is the pivot number

	X	Y	S_1	S_2	P	Solution	
Y	0.625	1	0.0625	0	0	375	$R_1/16$
S_2	10	8	0	1	0	3,600	
P	– 4,000	– 6,000	0	0	1	0	

Under table 3, make all the elements below 1 in the Y column to be zero using row elementary operation

	X	Y	S_1	S_2	P	Solution	
Y	0.625	1	0.0625	0	0	375	
S_2	5	0	– 0.5	1	0	600	$-8R_1 + R_2$
P	– 250	0	375	0	1	2,250,000	$6{,}000R_1 + R_3$

Under table 4, start again with the most negative number in the last row and look for the pivot number which will be 5. This is because 600/5=120 and 375/0.625=600

	X	Y	S_1	S_2	P	Solution	
Y	0.625	1	0.0625	0	0	375	
X	1	0	– 0.1	1	0	120	$R_2/5$
P	– 250	0	375	0	1	2,250,000	

Under table 5, make all the elements above and below 1 in the X column to be zero other

	X	Y	S_1	S_2	P	Solution	
Y	0	1	0.125	0.5	0	300	$-0.625R_2 + R_1$
X	1	0	– 0.1	0.2	0	120	
P	0	0	350	50	1	2,280,000	$250R_2 + R_3$

Decision: 120 carpets of X and 300 carpets of Y should be produced to maximize a profit of FRW 2,280,000.

iii)Areas of application linear programing

- Budgeting: Linear programming is useful in budgeting. When there is more than one scarce resource, linear programming can be used to identify the most profitable use of resources.
- Farming: Farmers use linear programming to increase the revenue of their operations, like what to grow, how much of it, and what to use it for.
- The Manufacturing Industry: Many other industries rely on linear programming to enhance the economy of their business.
- Capital budgeting: Linear programming can be used to determine the combination of investment proposals that should be selected if investment funds are restricted in more than one period.
- And any other related application not stated may be considered

QUESTION TWO

Marking Guide	*Marks*
a) (i) Definition of sampling techniques ((1 Mark each, max 2)	**2**
(ii) Advantages and disadvantages of stratified sampling	
Advantages (1 Mark each, max 2)	2
Disadvantages (1 Mark each, max 2)	2
Maximum marks	**4**
b) (i) Frequency distribution Table	
Cumulative frequency (0.5 marks each value, max 3)	3
Class boundary (0.5 marks each class, max 3)	3
Class mid-point (0.5 marks each value, max 3)	3
Maximum marks	**9**
(ii) Calculation of Σf and $\Sigma f x$ (0.5 mark each, max 1)	1
Formula for mean	0.5
Computation of mean	0.5
(iii) Formula for median	0.5
Showing median class	1
Computation of median	1.5
Maximum marks	**5**
Total marks	**20**

Model Answers

a) i) Differentiation of stratified and systematic sampling techniques

Stratified sampling technique is a sampling technique applied to a heterogeneous (having groups with varied attributes) population that can be subdivided into homogeneous groups known as strata. Stratified sampling involves obtaining simple samples from each of the strata of the population and the simple samples combined to give the stratified sample of the population. This ensures that each homogeneous segment of the population is proportionally represented in the sample

While Systematic sampling is a technique used when the population is listed in a given order or some of it is physically in evidence, for example, a row of houses or clients visiting a particular webpage in one hour on a given day.

The sample is drawn according to some predetermined point (place/object) chosen at random, and then systematically sample the nth item in the population, the number n chosen depending on the size of the sample required.

For a population of 100 items, if a sample of five items is required the n becomes 20, that is, the 20th, 40th, 60th, 80th and 100th items form the sample.

ii) Advantages and disadvantages of stratified sampling techniques

Advantages of systematic sampling

- It is easy to implement.
- Works where there is no sampling frame as long as the items are physically in evidence.
- Saves time and is not costly.

Disadvantages of systematic sampling

- Bias may occur where recurring sets in a population are possible.
- This sampling technique is not perfectly random. Once the starting point has been determined, all the subjects are predetermined.

b)

i) Raw data

18	20	19	21	35	27	36	40	20	37
25	21	28	42	30	31	28	26	33	18
36	26	21	30	21	30	24	22	23	19
32	23	20	36	26	37	25	20	22	25
30	23	26	45	28	27	30	27	29	27
26	24	25	21	39	26	28	30	28	30
27	28	36	24	26	21	21	25	37	33
21	20	34	33	30	29	28	20	23	24

Frequency Distribution Table

Class limits	Frequency (F)	Class boundaries	Class mid-point (x)	Cumulative Frequency (CF)	FX
18 – 22	20	17.5 – 22.5	20	20	400
23 – 27	25	22.5 – 27.5	25	45	625
28 – 32	19	27.5 – 32.5	30	64	570
33 – 37	12	32.5 – 37.5	35	76	420
38 – 42	3	37.5 – 42.5	40	79	120
43 – 47	1	42.5 – 47.5	45	80	45
	$\Sigma f = 80$				$\Sigma f x = 2180$

ii) Mean age

$$\text{Mean} = \frac{\Sigma f x}{\Sigma f} = \frac{2180}{80} = 27.25$$

iii) Median age

$$\text{Median} = L + \left(\frac{\frac{N}{2} - CFbmc}{Fmc}\right) \times c$$

L is lower class boundary of median class, L = 22.5

CFbm, cumulative frequency before median class, = 20

Fmc, frequency of median class, = 25,

C, class interval, = 5. N, the total frequency, = 80

$$\text{Median} = 22.5 + \left(\frac{\frac{80}{2} - 20}{25}\right) \times 5$$

$$\text{Median} = 22.5 + \left(\frac{20}{25}\right) \times 5$$

Median = 26.5

QUESTION THREE

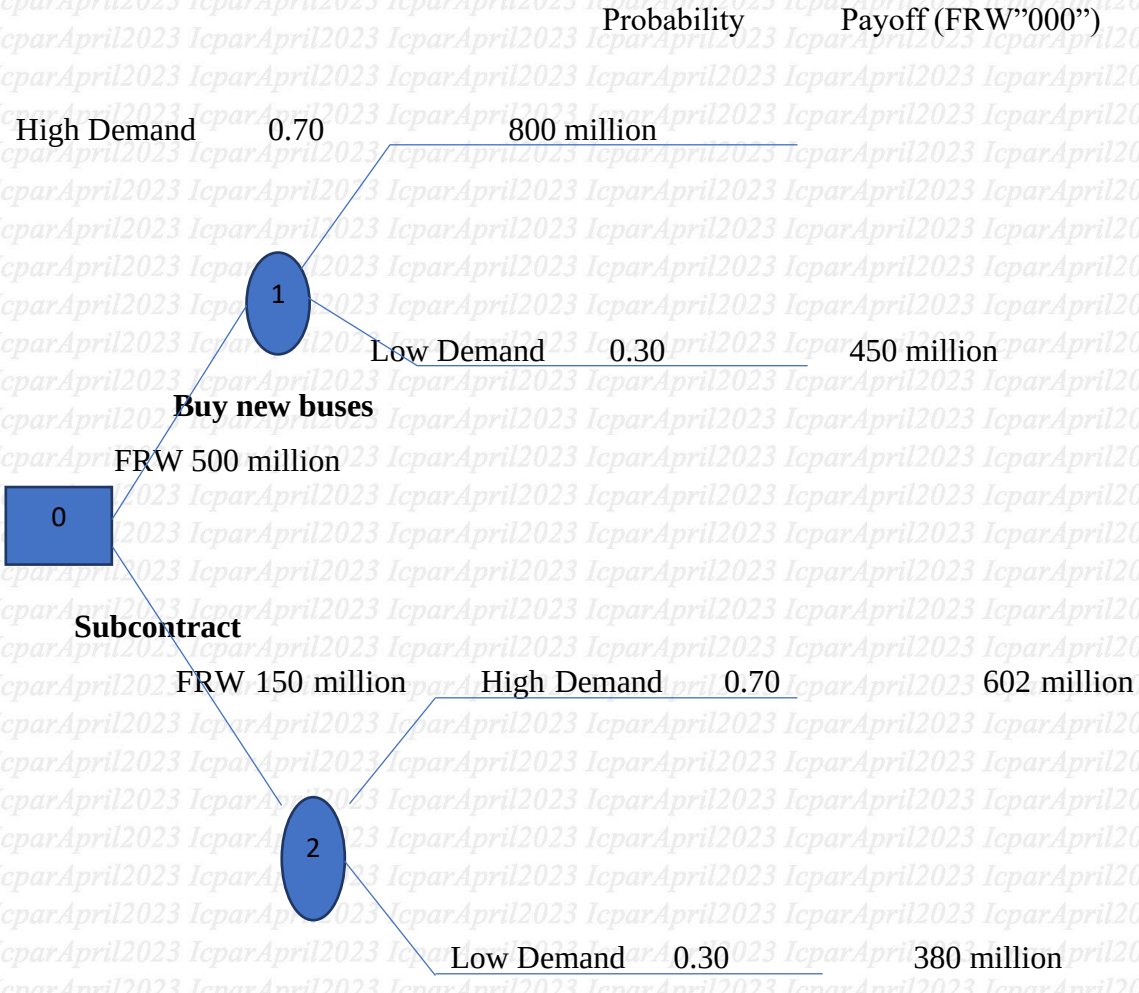
Marking Guide	*Marks*
a) (i) Branches of the decision tree (0.5 marks each, max 3)	3
Labels on the branches (0.5 marks each, max 3)	3
Maximum marks	**6**
(ii) Expected Monetary Value	
Expected monetary value of buying new buses	1.5
Expected monetary value of sub contraction	1.5
Maximum marks	**3**
(iii) Advice on the right decision to make	**1**
b) i) Decision under maximax criterion	**1**

ii) Computation of Laplace payoffs (1 Mark each, max 3)	3
Decision about Laplace criterion	1
Maximum marks	**4**
iii) Minimax criterion	
Calculation regret for favorable condition (0.5 Marks each)	1.5
Calculation regret for stable condition (0.5 Marks each)	1.5
Calculation regret for unfavorable condition (0.5 Marks each)	1.5
Decision under Minimax criterion	0.5
Maximum marks	**5**
Total marks	**20**

Model Answers

a) i) Decision tree

Decision Tree



ii) Expected Monetary Value (EMV)

Buy new bus: EMV = [(0.7*800 M) + (0.3*450 M)]

= FRW 560 M + FRW 135 M = FRW 695 M.

EMV after subtracting initial cost = FRW 695 M - FRW 500 M = FRW 195 M

Sub contraction: EMV = [(0.7*602M) + (0.3*380M)]

= FRW 421.4M + FRW 114M = FRW 535.4M

EMV after subtracting initial cost = FRW 535.4M - FRW 150M = FRW 385.4M

iii) The right decision to make is to choose the second possibility of sub contraction

b)

Alternative projects	**Market Conditions**			
	Favorable	**Stable**	**Unfavorable**	**Maximum payoffs**
A	71	60	46	71
B	67	54	41	67
C	76	65	51	**76**

i) **Maximax criterion**

Decision under Maximax is to choose project C because it has the maximum of the maximum payoff of FRW76 million

ii) **Laplace criterion**

Project A; Payoff P = (71 + 60 + 46)/3 = 177/3 = FRW 59 million

Project B; Payoff P = (67 + 54 + 41)/3 = 162/3 = FRW 54 million

Project C; Payoff P = (76 + 65 + 51)/3 = 192/3 = FRW 64 million

Decision under Laplace is to choose project C because it has the highest payoff of FRW64 million

iii)**Minimax criterion**

Regret table

Alternative projects	**Market Conditions**			
	Favorable	**Stable**	**Unfavorable**	**Maximum regrets**
A	16	0	5	16
B	9	17	10	17
C	0	6	0	**6**

Decision under minimax is to choose project C because it has the minimum of the maximum regret of FRW6 million

QUESTION FOUR

Marking Guide	*Marks*
a) Statement of the hypotheses (0.5 Marks each, max 1)	1
Computation of expected frequencies (0.5 Marks each, max 3)	3
Computation of $(O-E)^2$ /E (1 Mark each, max 6)	6
Calculated χ^2	0.5
Tabulated χ^2 (reading from the table)	0.5
Conclusion	1
Maximum marks	**12**
b) Statement of the hypotheses (0.5 Marks each, max 1)	1
Finding the critical value of t from the t table	1
Calculation of the standardized value of t	2
Conclusion	1
Maximum marks	**5**
c) Applications of Poisson distribution (1 Mark each, max 3)	3
Total marks	**20**

Model Answers

a)

Level	**Male**	**Female**	**Total**
Foundation	120	140	**260**
Intermediate	90	110	**200**
Advanced	60	80	**140**
Total	**270**	**330**	**600**

Solution:

H_0: there is no difference between male and female in passing CPA exams

H_1: there is a difference between male and female in passing CPA exams

Calculation of expected frequencies (E)

$$E_{11} = \frac{R_1 C_1}{n} = \frac{260 \times 270}{600} = 117$$

$$E_{12} = \frac{R_1 C_2}{n} = \frac{260 \times 330}{600} = 143$$

$$E_{21} = \frac{R_2 C_1}{n} = \frac{200 \times 270}{600} = 90$$

$$E_{22} = \frac{R_2 C_2}{n} = \frac{200 \times 330}{600} = 110$$

$$E_{31} = \frac{R_3 C_1}{n} = \frac{140 \times 270}{600} = 63$$

$$E_{32} = \frac{R_3 C_2}{n} = \frac{140 \times 330}{600} = 77$$

$\chi^2 = \sum \frac{(O-E)^2}{E}$

Observed values (O)	Expected Frequency (E)	$(O-E)^2$	$(O-E)^2$/E
120	117	9	0.077
140	143	9	0.063
90	90	0	0
110	110	0	0
60	63	9	0.143
80	77	9	0.117
			$\sum \frac{(O-E)^2}{E} = 0.4$

Calculated $\chi^2 = 0.4$

Df = (r – 1) (c – 1) = (3 – 1) (2 – 1) = 2

Df is degree of freedom

R is number of row

C is number of column

Tabulated $\chi^2_{(0.05)} = 5.99$

Since calculated χ^2 < tabulated $\chi^2_{(0.05)}$, H_0 is accepted 0.4< 5.99

Conclusion: There is no difference between male and female in passing CPA exams. The pass rate is independent of gender.

b) 1. Statement of hypotheses

Null hypothesis, H_0: μ (mean weight) = 3.5 kgs

Alternative hypothesis, H_A: μ (mean weight) < 3.5 kgs

2. The level of significance is 1% and critical value of one tailed test under degree of freedom (n-1) of 24 is 2.492. Critical value of t is 2.492

3. The standardizes value of the sample mean is

Sample mean, $\bar{x}$ = 3.0kgs, standard deviation, s = 0.5 kgs sample size, n = 25, Population mean, μ = 3.5 kgs

$t = \left|\frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}\right| = \left|\frac{3.5-3.0}{\frac{0.5}{\sqrt{25}}}\right|$ = 5. Standardized value of t is 5

6. Conclusion: Since the standardized value of 5 is greater than the critical value of 2.492, we accept the null hypothesis but reject the alternative hypothesis at 1% level of significance. This means that babies are born with weight of 3.5 kgs and therefore the belief that babies are with the weight of 3.5 kgs is true in Mahoro Cell

c) Application of Poisson distribution in analysis of Poisson events

- Car accidents on a particular road in one day.
- Telephone calls made to a switch board in a given minute.
- Accidents in a factory in one week.
- Insurance claims made to a company in a given time.
- Printing mistakes in a page of a book
- Quality control departments of manufacturing industries to count the number of defects found in a lot
- It is also used in queuing theory

QUESTION FIVE

Marking guide	*Marks*
i) Computation of weighted indices in the table (0.5 marks each, max 12)	12
Laspeyre's price index (0.5 Marks for formula and 0.5 Marks for computation)	1
Paasche's price index (0.5 Marks for formula and 0.5 Marks for computation)	1
Marshall Edge Worth's price index (0.5 Marks for formula and 0.5 Marks for computation)	1
Maximum marks	**15**
ii) Interpretation (1 Mark each, max 2)	2
iii) Problems of constructing index numbers (1 Mark each, max 3)	3
Maximum marks	**5**
Total marks	**20**

Model Answers

i)

	2020		**2022**					
Item	Price (FRW)/ Kg - **P0**	Quantity (Kgs) - **Q0**	Price (FRW)/Kg - **P1**	Quantity (Kgs) - **Q1**	**P0Q0 ("000")**	**P1Q1 ("000")**	**P1Q0 ("000")**	**P0Q1 ("000")**
Sugar	1,000	600	1,200	500	600	600	720	500
Rice	1,500	100	1,700	150	150	255	170	225
Salt	200	160	250	200	32	50	40	40
Beans	600	350	800	460	210	368	280	276
Potatoes	250	420	400	500	105	200	168	125
					1,097	**1,473**	**1,378**	**1,166**

Laspeyre's price index

Laspeyre's price index $= \frac{\sum P_1Q_0}{\sum P_0Q_0} \times 100 = \frac{1,378,000}{1,097,000} \times 100 = 125.62$

Paasche's price index

Paasche's price index $= \frac{\sum P_1Q_1}{\sum P_0Q_1} \times 100 = \frac{1,473,000}{1,166,000} \times 100 = 126.33$

Marshall Edge Worth's price index

Marshall Edge Worth's price index $= \frac{\sum P_1Q_0 + \sum P_1Q_1}{\sum P_0Q_0 + \sum P_0Q_1} \times 100 = \frac{1,378,000 + 1,473,000}{1,097,000 + 1,166,000} \times 100 = 125.98$

ii) **Interpretation**

For Laspyere's price index: There was an increase of 25.62 % for the basket of commodities from 2020 to 2022.

For Paasche's price index: There was an increase of 26.33 % for the basket of commodities from 2020 to 2022

For Marshall Edge Worth's price index: There was an increase of 25.98 % for the basket of commodities from 2020 to 2022

iii) Problems of constructing index numbers:

1. It is not possible to make comparisons between different locations:

Even if various locations within a country are chosen, the same index number cannot be assigned to them. This is due to variances in people's consumption habits. Individuals in the northern part of country consume different commodities than people in the southern portion of same country.

2. Not Appropriate to Individuals:

An index number is not applicable to a single person who is a member of the group it was created. A person may not be affected if there is a rise in the price level index number shows. This is since an index number reflects averages.

3. Difficulty in Choosing a Statistical Approach:

Another challenge is deciding on a suitable approach for calculating averages. However, each strategy produces a unique set of findings. As a result, deciding which strategy to use is challenging.

4. Difficulties Resulting from Changes Over Time:

In today's world, changes in commodities occur on a continuous basis because of technological advancements. So, consumers begin to consume them, and instead of the old, new commodities are introduced. Furthermore, commodity prices may fluctuate because of technological advancements. They might fall. However, when calculating the index numbers, new commodities are not added to the list of commodities. As a result, the index figures based on ancient commodities are unreal.

QUESTION SIX

Marking guide	*Marks*
a) i) Derivative of revenue function	1
Second derivative of revenue function	1
Total revenue at the given point	1
Maximum marks	**3**
ii) Deriving Marginal cost function	1
First derivative of marginal cost function	1
Second derivative of marginal cost function	1
Marginal cost at minimum level	1
Maximum marks	**4**
iii) Deriving the profit function	1
First derivative of profit function at each step (0.5 Marks each, max 3.0)	3
Second derivative of profit function at each step (0.5 Marks each, max 2.0)	2
Maximum profit	1
Maximum marks	**7**
b) i) Finding the profit function	1
ii) Calculation of break – even point	2
Interpretation of break – even points	1
iii) Computation of number of shoes and maximum profit	
1 Mark for Formula and 1 Mark for computation	2
Total marks	**20**

Model Answers

a) i) Output when revenue is maximum

Revenue function R(x) = $21x - x^2$

Revenue is maximum at the second derivative when the derivative < 0

$\frac{dR(x)}{dx} = \frac{d(21x-x^2)}{dx} = 21 - 2x$

$\frac{dR(x)}{dx} = 0, 21 - 2x = 0$

x = 21/2 = 10.5

Find the second derivative for confirmation of the sign

$\frac{d^2R(x)}{dx^2} = \frac{d^2(21-2x)}{x^2} = -2$

$\frac{d^2R(x)}{dx^2} < 0$, therefore revenue is maximum at this point

Total revenue at this point

The output that maximizes revenue is x = 10.5. It is this x = 10.5 that is substituted into the revenue function to find the total revenue

Revenue function R(x) = $21x - x^2$

Revenue function R(x) = $21(10.5) - (10.5)^2$ = 220.5 – 110.25 = FRW 110.25 million

ii) Marginal Cost at a minimum level

Total cost function C = $\frac{x^3}{3} - 3x^2 + 9x + 16$

First find marginal cost from the derivative of total cost function

$\frac{dC(x)}{dx} = \frac{d(\frac{x^3}{3}-3x^2+9x+16)}{dx} = x^2 - 6x + 9$

Marginal cost MC = $x^2 - 6x + 9$

Find the first derivative of marginal cost to get the output that maximizes MC

$\frac{dMC(x)}{dx} = \frac{d(x^2-6x+9)}{dx} = 2x - 6$

$\frac{dMC(x)}{dx}$ = 0, $2x - 6 = 0$. x = 3

Then find the second derivative of MC to check for the sign. If it is found to be positive then MC is minimum

$\frac{d^2MC(x)}{dx^2} = \frac{d^2(2x-6)}{x^2} = 2$

$\frac{d^2MC(x)}{dx^2} > 0$, therefore MC is minimum at this point

Marginal cost MC(x) = $x^2 - 6x + 9$

Marginal Cost when x =3. MC = $(3)^2 - 6(3) + 9 = 0$

The Marginal cost at minimum level is 0

iii) Output that maximizes profit

Profit function (P) = Total Revenue (R) – Total Cost (C)

Profit function (P) = $(21x - x^2) - (\frac{x^3}{3} - 3x^2 + 9x + 16)$

Profit function (P) = $-\frac{x^3}{3} + 2x^2 + 12x - 16$

Find the first derivative of profit

$\frac{dP(x)}{dx} = \frac{d(-\frac{x^3}{3} + 2x^2 + 12x - 16)}{dx} = -x^2 + 4x + 12$

$\frac{dP(x)}{dx} = 0, -x^2 + 4x + 12 = 0$

$x^2 - 4x - 12 = 0$

$x^2 - 6x + 2x - 12 = 0$

$x(x - 6) + 2(x - 6) = 0$

$(x - 6) = 0$ or $(x + 2) = 0$

$x = 6$ or $x - 2$

Proceed to find the second derivative

$\frac{d^2P(x)}{dx^2} = \frac{d^2(-x^2+4x+12)}{x^2} = -2x + 4$

$\frac{d^2P(x)}{dx^2} = -2x + 4$

At $x = 6$, $\frac{d^2P(x)}{dx^2} = -(2 \times 6) + 4 = -8 < 0$

At $x = -2$, $\frac{d^2P(x)}{dx^2} = -(2 \times -2) + 4 = 8 > 0$

Therefore, profit is maximum at $x = 6$

Then compute the maximum profit using the output of 6 units

Profit function (P) = $-\frac{x^3}{3} + 2x^2 + 12x - 16$

Profit (P) = $-\frac{(6)^3}{3} + 2(6)^2 + (12 * 6) - 16 = 56$

Maximum Profit = FRW 56 million

b) i) Profit function

Profit = Total Revenue - Total Cost

Profit = $[(30 - 2x)x] - [2x + 26]$

Profit = $30x - 2x^2 - 2x - 26$

Profit function = $28x - 2x^2 - 26$ for $0 \leq x \leq 15$

ii) Break even points and interpretation

Break-even point is obtained when profit is 0

$28x - 2x^2 - 26 = 0$ divide through by 2

$14x - x^2 - 13 = 0$ rearranging to find the values of x

$x^2 - 14x + 13 = 0$ by factorization

$x^2 - x - 13x + 13 = 0$

$x(x-1) - 13(x-1) = 0.\ (x-1) = 0$ or $(x-13) = 0$

$x = 1$ or $x = 13$

The break – even points are at $x = 1$ and $x = 13$

Interpretation: The firm should produce between one 1 or 13 shoes to break - even

iii) Number of shoes which need to be sold in order to maximize profit

We can find the first and second derivatives of the profit function or use the vertex formula since the profit function is a quadratic equation

Profit $=28x - 2x^2 - 26$

$y = ax^2 + bx + c$

$V\left(-\frac{b}{2a}, -\frac{\Delta}{4a}\right)$ *where* $\Delta = b^2 - 4ac$, *V is vertex, a =-2, b =28 and c =-26*

$V\left(-\frac{28}{2(-2)}, -\frac{28^2-4*(-2)*(-26)}{4*(-2)}\right)$

$V(7, 72)$

7 shoes are needed to maximize a profit of FRW 72,000

QUESTION SEVEN

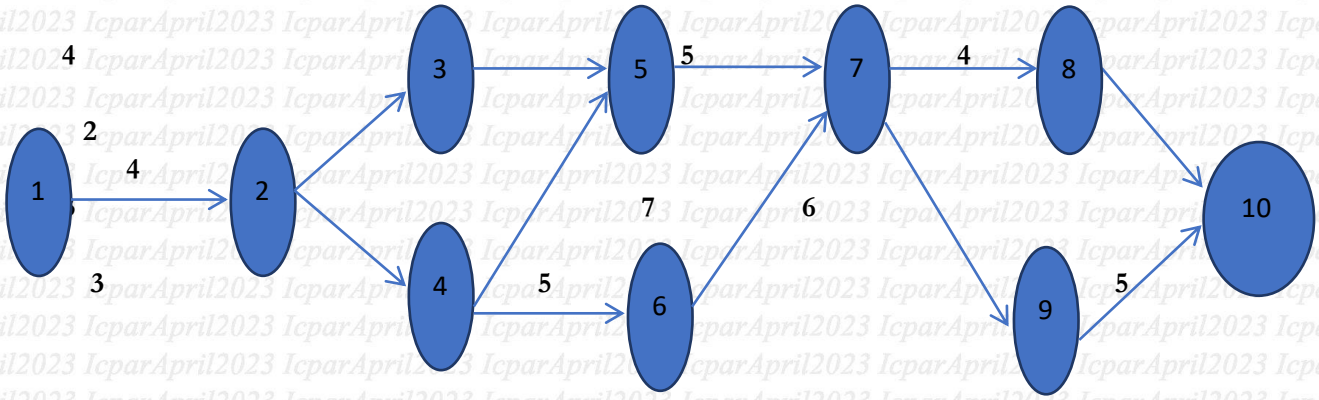
Marking Guide	***Marks***
a) Computation expected duration (0.5 Marks each, max 6)	6
b) Construction of each activity (0.5 Marks each, max 6)	6
c) Identification of the critical path	1
d) Calculation of the variance of each critical activity (0.5 Marks each, max 3)	3
Calculation of total variance	0.5
Calculation of standard deviation	0.5
e) Computation of probability of completion	
Formula	1
Computation	2
Total marks	**20**

Model Answer

a) Expected time

Activity	Optimistic Time (in days)	Most Likely Time (in days)	Pessimistic Time (in days)	Expected Tme (in days) - Te=(to+4tm+tp)/6	Variance (in days)σ2 = [(tp - to)/6]^2
(1 – 2)	2	4	6	4	0.444
(2 – 3)	1	2	3	2	
(2 – 4)	1	3	5	3	0.444
(3 – 5)	3	4	5	4	
(4 – 5)	2	3	4	3	
(4 – 6)	3	5	7	5	0.444
(5 – 7)	4	5	6	5	
(6 – 7)	6	7	8	7	0.111
(7 – 8)	2	4	6	4	
(7 – 9)	5	6	7	6	0.111
(8 – 10)	1	2	3	2	
(9 – 10)	3	5	7	5	0.444
Sum					**1.998**

b) Network diagram



c) Identification of the critical path

1 – 2 – 3 – 5 – 7 – 8 – 10 Duration: 4 + 2 + 4 + 5 + 4 + 2 = 21weeks

1 – 2 – 3 – 5 – 7 – 9 – 10 Duration: 4 + 2 + 4 + 5 + 6 + 5 = 26 weeks

1 – 2 – 4 – 5 – 7 – 8 – 10 Duration: 4 + 3 + 3 + 5 + 4 + 2 = 21weeks

1 – 2 – 4 – 5 – 7 – 9 – 10 Duration: 4 + 3 + 3 + 5 + 6+ 5 = 26 weeks

1 – 2 – 4 – 6 – 7 – 8 – 10 Duration: 4 + 3 + 5 + 7 + 4 + 2 = 25 weeks

1 – 2 – 4 – 6 – 7 – 9 – 10 Duration: 4 + 3 + 5 + 7 + 6 + 5 = 30 weeks Critical path

d) Variance and standard deviation

Variance = 0.44+0.44+0.44+0.44+0.11+0.11 = 1.98

Standard deviation = $\sqrt{1.98}$ = 1.41

e) Probability of completing the project within 28 weeks or less

Z = $\frac{Ts-Te}{\sigma}$ Z is the z score for finding the probability in question

Ts is the scheduled project duration

Te is expected time/critical path duration

P($Z \leq \frac{28-30}{1.41}$) = - 1.42)

Reading from the table the P-value of z = - 1.42 is 0.0778 ≈7.78%.

END OF MARKING GUIDE AND MODEL ANSWER